Common valuations of division polynomials

Bartosz Naskręcki
Uniwersytet im. Adama Mickiewicza w Poznaniu
e-mail: bartosz.naskrecki@amu.edu.pl

In this talk we discuss the results of joint work with Matteo Verzobio [1] about the common valuation of the division polynomials of points on elliptic curves. We prove a formula for the cancellation exponent between division polynomials psi and phi associated with a sequence of points on an elliptic curve defined over a discrete valuation field. The formula is identical with the result of Yabuta-Voutier [2] for the case of finite extension of p-adic field $\mathbb{Q}_p$ and generalizes to the case of non-standard Kodaira types for non-perfect residue fields. Our proof applied to the case of $\mathbb{Q}_p$ is much shorter and depends exclusively on the elementary properties of the Néron local heights.The study of the cancellation sequence has some interesting applications. For example, knowing the behaviour of it was necessary to show that every elliptic divisibility sequence satisfies a recurrence relation originally defined by Ward.

[1] B. Naskręcki, M. Verzobio, Common valuations of division polynomials, *Proc. A. R. Soc. Edinb.* , 2024, DOI:10.1017/prm.2024.7, 1–15

[2] P. Voutier and M. Yabuta, The greatest common valuation of ϕ_n and ψ_n at points on elliptic curves, *J. Number. Theory*, 229 (2021), 16–38.